Mental strategies – progression document

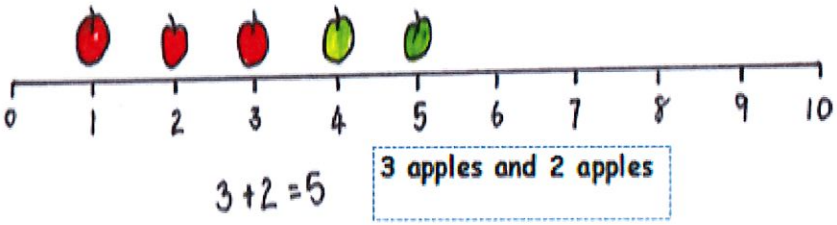
This document should be read alongside the Wootton Calculation Progression document.

Whilst this document focuses on the number line as the main representation, it is still *crucial* that pupils are introduced to a range of representations of core concepts alongside this.

Addition

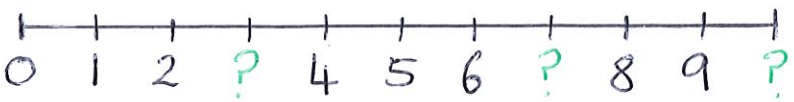
YR:

Exposure to the number line as a representation of numbers to 10 (and 20), **alongside** practical resources and other pictorial illustrations.

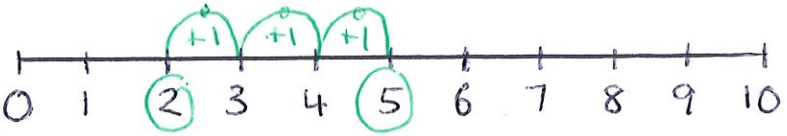


Build life-size number lines and practise finding numbers and jumping forwards and counting along the line.

Find a given number on a number line. Fill in missing numbers on a number line.



Count forwards on a number line. - vary language used.



Start at 2.
Count on 3 (jumps).
Where do you land?

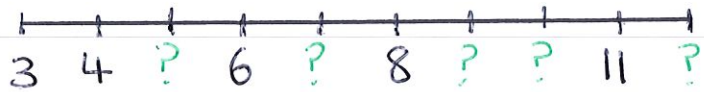
Embed number bonds within 5 using a range of representations including the number line.

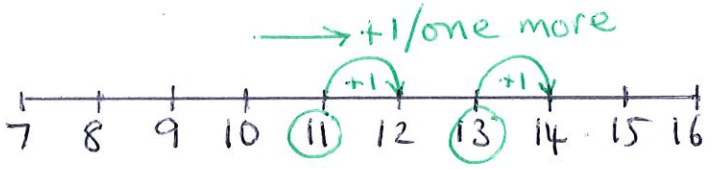
Write simple addition sentences: 3 + 4 = ☐

Y1:

Extend place value to 20 (and beyond).

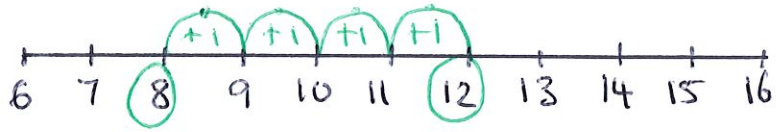
Locate a given number on a number line. Fill in missing numbers on a number line. Find one more using a number line.



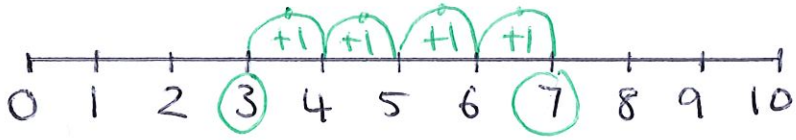


- One more than 11?
- 13 + 1 = ?

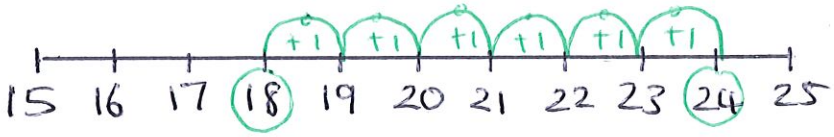
Count forwards in ones on a number line and record this using "jumps". Begin to link this to simple addition calculations within 10, then crossing the tens boundary, then across 20.



- Start on 8. Count on 4 jumps. Where do you land?

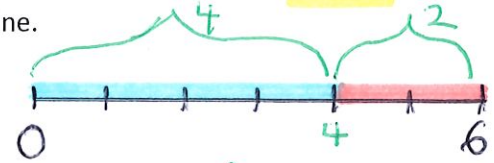


- 3 + 4 = ?

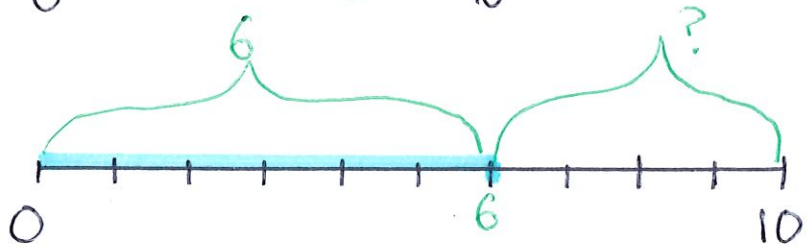


- 18 + 6 = ?

Embed number bonds for 6, 7, 8, 9 and 10 using a range of representations including the number line.



4 + 2 = 6

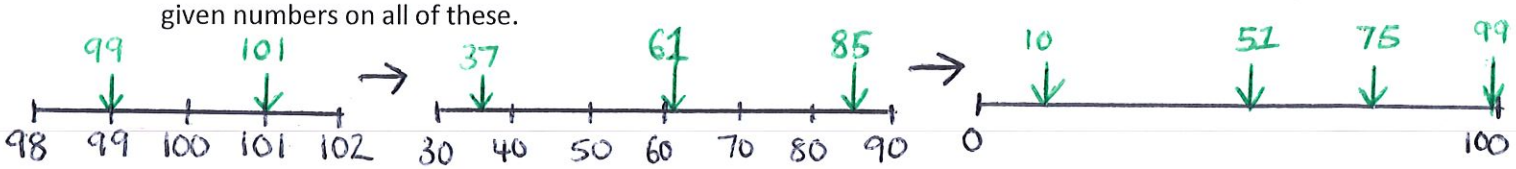


6 + □ = 10?

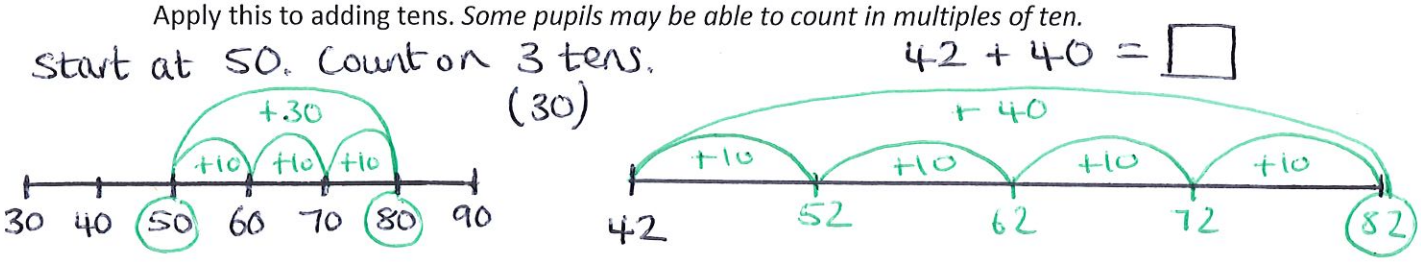
Potential progression for pupils working at a Greater Depth from a structured number line, to a number line labelled in multiples of ten, to a blank number line.

Y2:

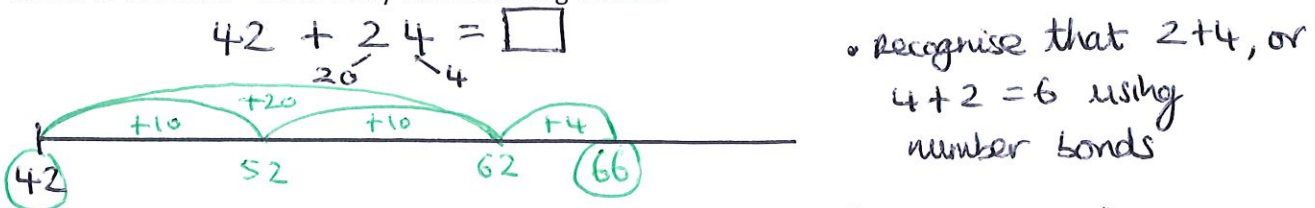
Extend place value to 100. Count in ones and tens across the 100 boundary. Progression from using a structured number line, to a number line labelled in multiples of ten, to a blank number line. Locate given numbers on all of these.



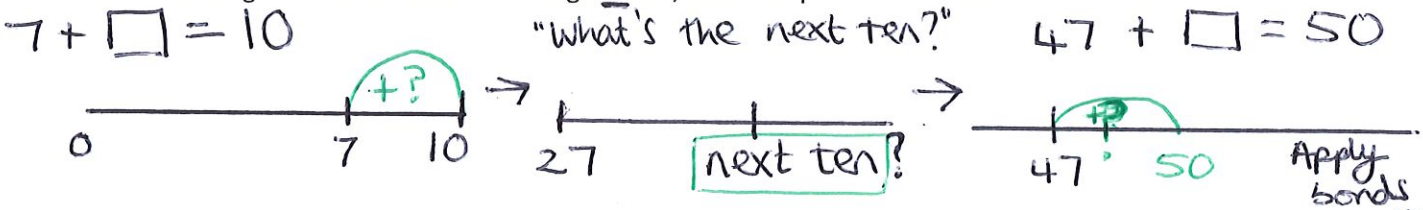
Count in 10s using a number line, from 0 and other multiples of 10, and then from any number. Apply this to adding tens. *Some pupils may be able to count in multiples of ten.*



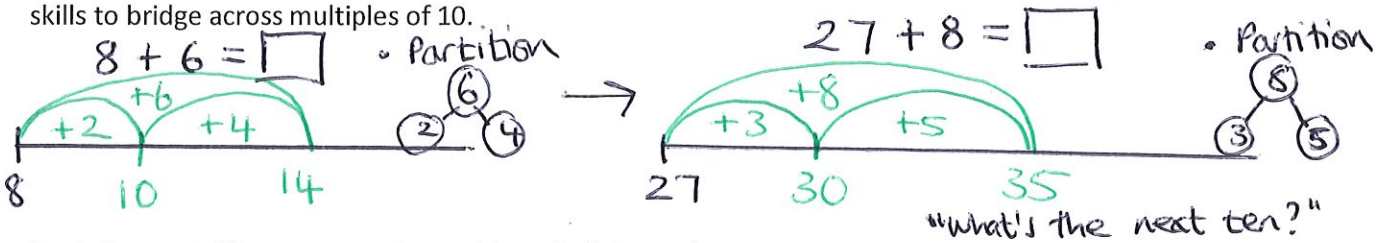
Add tens and ones (not crossing tens boundaries) by partitioning the second number. Apply number bonds to add ones – move away from counting in ones.



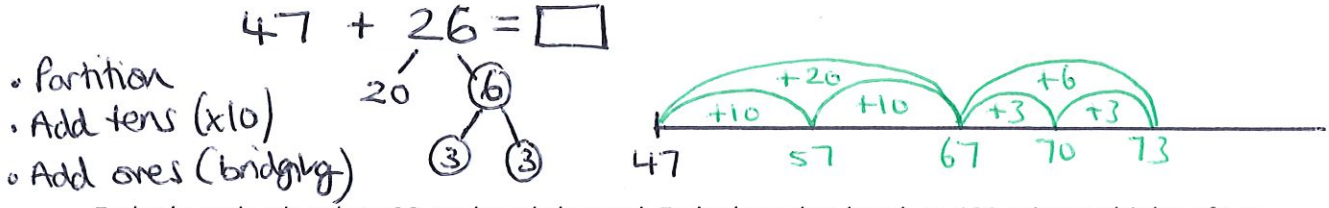
Use knowledge of number bonds to bridge to ten, and multiples of 10 - "what's the next ten?"



Apply number bonds within 10 to bridge across ten, partitioning e.g. 8 into 3 and 5, 4 and 4, 2 and 6, or 7 and 1 as necessary. *It is crucial that number bonds to 6, 7, 8, and 9 are secure*. Use the same skills to bridge across multiples of 10.

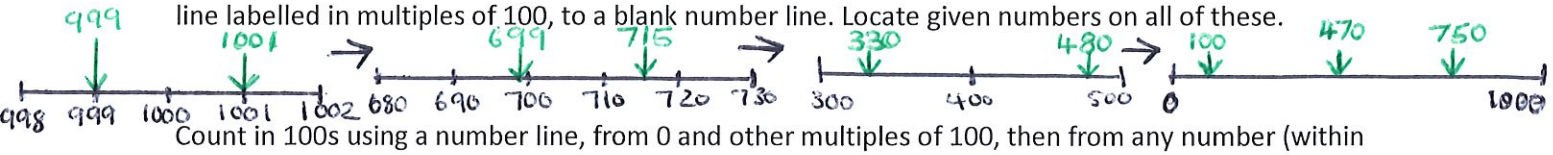


Apply learned skills concurrently to add any 2-digit numbers.

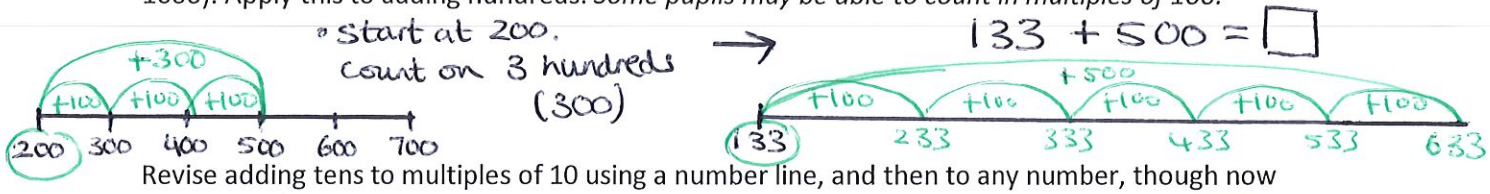


Embed number bonds to 20, and apply beyond. Embed number bonds to 100 using multiples of ten. Some children may be able to apply these in tandem to work out number bonds to 100 using any 2-digit number. Some pupils may be able to apply their skills to cross the 100 boundary.

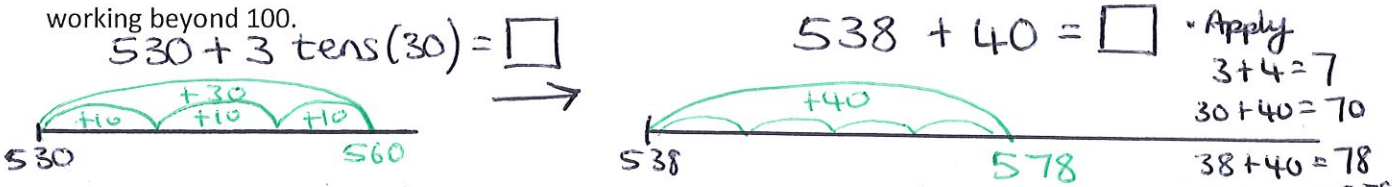
Y3: Extend place value to 1000. Count in ones, tens and 100s across all 100s boundaries, and across the 1000 boundary. Progression from using a number line labelled in multiples of 10, to a number line labelled in multiples of 100, to a blank number line. Locate given numbers on all of these.



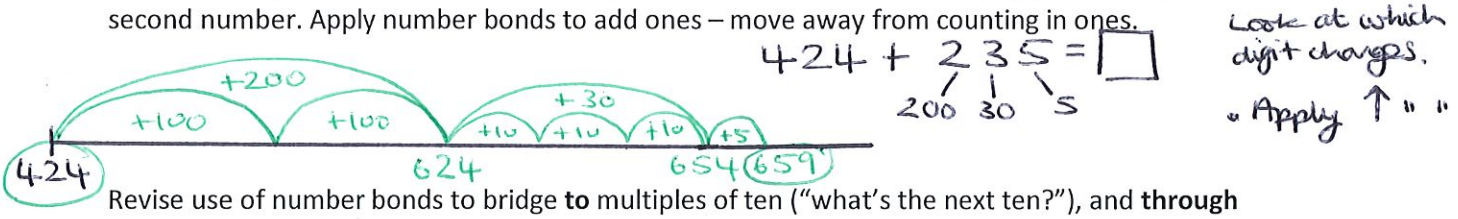
Count in 100s using a number line, from 0 and other multiples of 100, then from any number (within 1000). Apply this to adding hundreds. *Some pupils may be able to count in multiples of 100.*



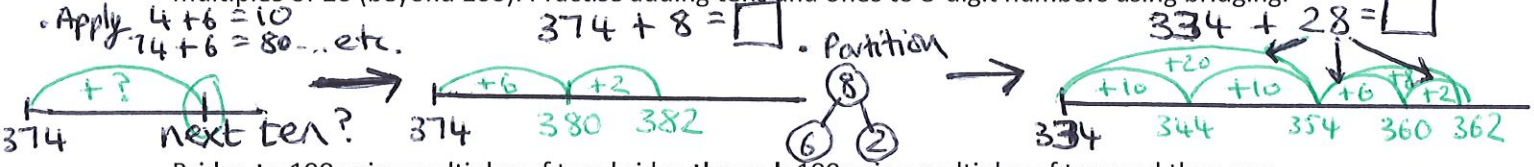
Revise adding tens to multiples of 10 using a number line, and then to any number, though now working beyond 100.



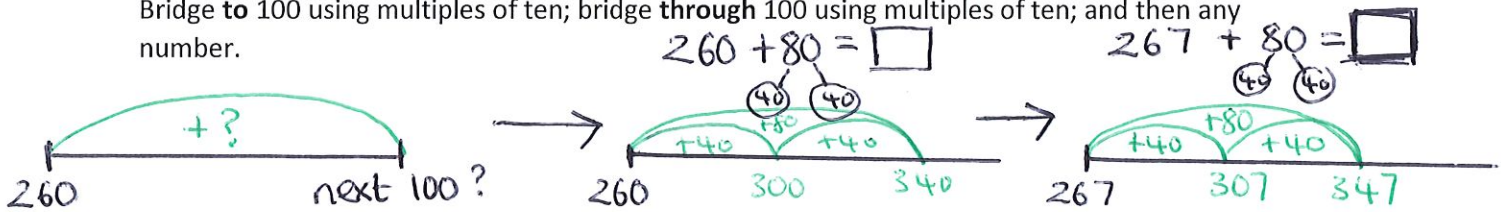
Add hundreds, tens and ones (not crossing tens and hundreds boundaries) by partitioning the second number. Apply number bonds to add ones – move away from counting in ones.



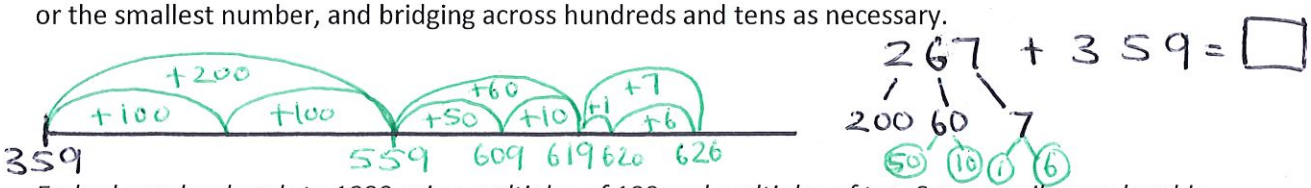
Revise use of number bonds to bridge **to** multiples of ten ("what's the next ten?"), and **through** multiples of 10 (beyond 100). Practise adding tens and ones to 3-digit numbers using bridging.



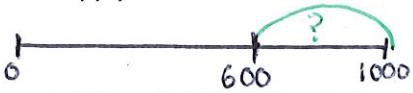
Bridge **to** 100 using multiples of ten; bridge **through** 100 using multiples of ten; and then any number.



Apply learned skills concurrently to add 3-digit numbers by partitioning either the second number, or the smallest number, and bridging across hundreds and tens as necessary.



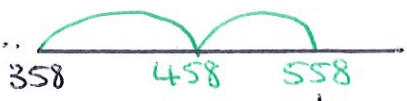
Embed number bonds to 1000 using multiples of 100 and multiples of ten. Some pupils may be able to apply these in tandem to work out number bonds to 1000 using any 3-digit numbers.



• Apply 6 + 4 = 10, 60 + 40 = 100, 600 + 400 = 1000

Additional skills: Estimate by rounding and adjusting.

358 + 199 = □ Round 199 to 200 ... ... minus 1 ... = 557



Apply mental bridging to the expanded column method using 3-digit numbers.

	300 +	50 +	8
+	100 +	90 +	7
	400 +	140 +	15

8 + 7 → mental bridging (8 5)

50 + 90 → mental bridging (10 40) = 555

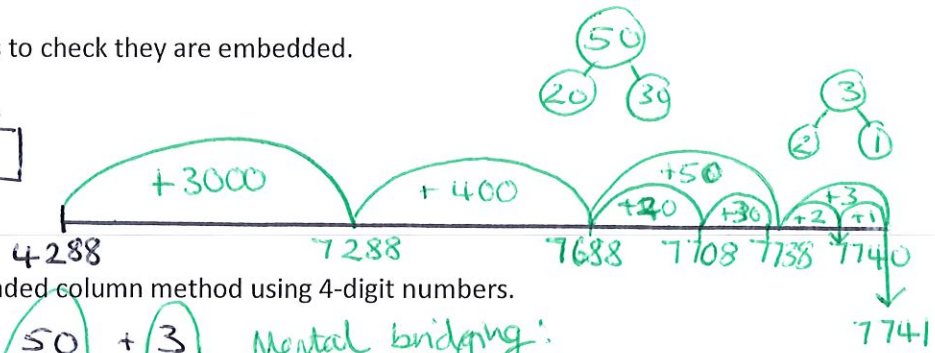
Y4:

Revise skills from prior year groups to check they are embedded.

Apply the same skills beyond 1000.

$4288 + 3453 = \square$

3000 400 50 3



Apply mental bridging to the expanded column method using 4-digit numbers.

	3000	+ 400	+ 50	+ 3
+	4000	+ 200	+ 80	+ 8
	7000	600	130	11

Mental bridging:

8 + 3 = 11 (2 1)

80 + 50 = 130 (20 30)

Progress to the formal column method (first without, and then with carrying). Mental bridging should now be fluent.

$$\begin{array}{r} 342 \\ +\ 123 \\ \hline 465 \\ \hline \end{array}$$

Ensure PV understanding is secure - what is the value of the circled digit? 60

$$\begin{array}{r} 359 \\ +\ 374 \\ \hline 733 \\ \hline \end{array}$$

Carry digits under the line. Cross out once added on.

Y5 and Y6:

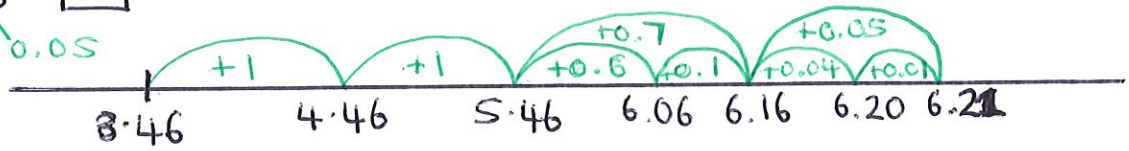
Revise skills from prior year groups to check they are embedded.

Apply the same skills to increasingly large numbers.

Apply the same skills to work with decimals, demonstrating understanding of the relative "sizes" or values of decimals.

$3.46 + 2.75 = \square$

2 0.7 0.05

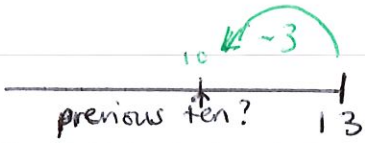


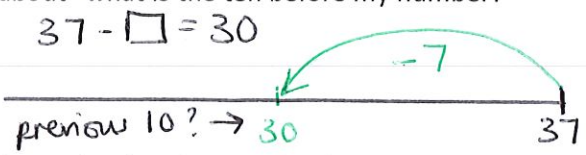
Select appropriate and efficient methods to solve problems.

Subtraction

All year groups: Repeat the processes described for Addition, but this time counting backwards for Subtraction. Embed subtraction facts, recognising fact families.

Y2: When bridging backwards **to** ten, think about "what is the ten before my number?"

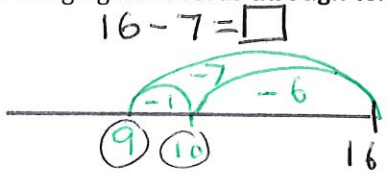


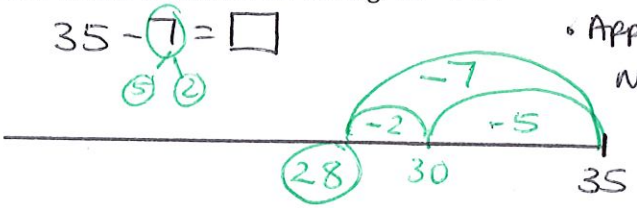


• Apply number bonds

When bridging backwards **through** ten, apply number bonds to and within 10. E.g. 35 – 7 = ?

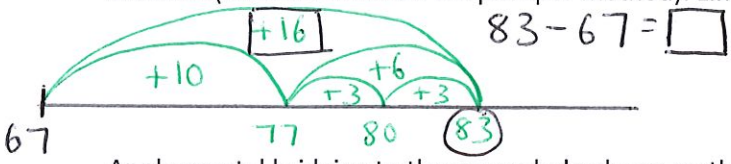
Y3:

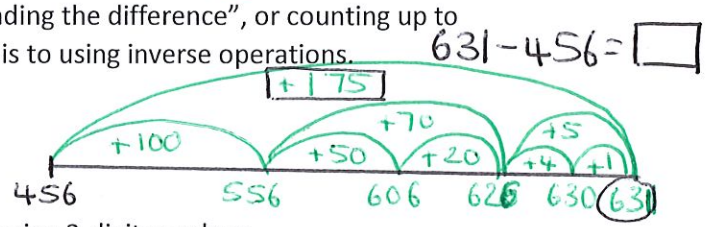




• Apply number bonds

Additional skills: once bridging is secured, introduce "finding the difference", or counting up to subtract (sometimes called shopkeeper method). Link this to using inverse operations.





Apply mental bridging to the expanded column method using 3-digit numbers.

Y4:

$$\begin{array}{r} 400 + 60 + 3 \\ -\ 100 + 20 + 2 \\ \hline 300 + 40 + 1 \end{array} = 341 \qquad \longrightarrow \qquad \begin{array}{r} 400\ \ 60\ \ 3 \\ -\ 100\ \ 20\ \ 2 \\ \hline 300\ \ 40\ \ 1 \end{array} = 341$$

Additional skills: Apply mental bridging to the expanded column method using 4-digit numbers, including exchanging.

$$\begin{array}{r} 5000\ \ 600\ \ 30\ \ 8 \\ -\ 2000\ \ 800\ \ 50\ \ 9 \\ \hline 2000\ \ 700\ \ 70\ \ 9 \end{array} = 2779$$

Progress to the formal column method (first without, and then with exchanging). Mental bridging should now be fluent.

$$\begin{array}{r} 9864 \\ -\ 1243 \\ \hline 8621 \end{array}$$

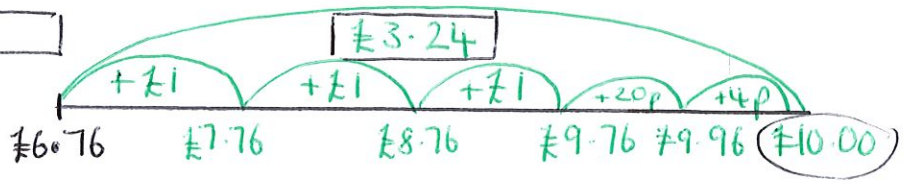
• ensure PV understanding is secure – what is the value of the circled digit?

$$\longrightarrow \quad \begin{array}{r} 4\ 15\ 13\ 8 \\ -\ 2852 \\ \hline 2786 \end{array}$$

Secure "finding the difference" as an efficient mental subtraction method.

Y5 & Y6:

£10.00 - £6.76 = □



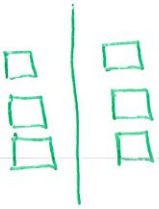
Repeat the processes described for addition with subtraction, including application to decimals.

Secure "finding the difference" as an efficient mental subtraction method.

Multiplication

YR: Double to solve problems through practical and visual doubling – e.g. resources, ladybird spots

2 children have 3 bricks each. How many altogether?



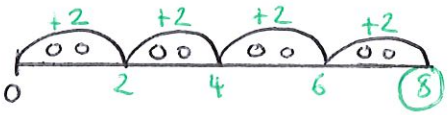
Double 3 =

Jack's beanstalk doubles in height. It started at 5 leaves tall. How many now?

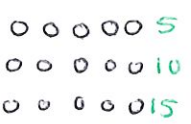
Y1: Jump in 2s, 5s and 10s on a structured number line, <u>alongside</u> arrays and drawn groups.

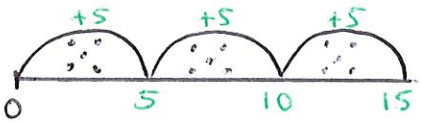
4 x 2 = □

(lots of) (groups of)



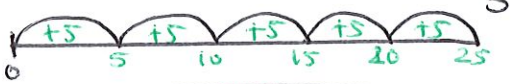
3 x 5 =



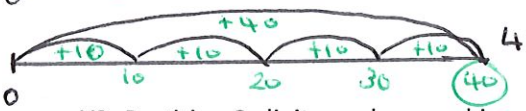


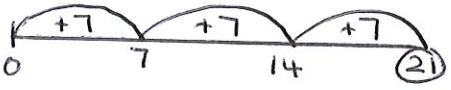
Y2: Jump in 2s, 5s and 10s confidently. *Some pupils may be able to count in multiples of 10.* Apply skills to other amounts, alongside arrays. Once recording is secure, move on to listing tables.

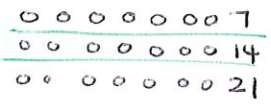
5 x 5 = □



4 x 10 = □





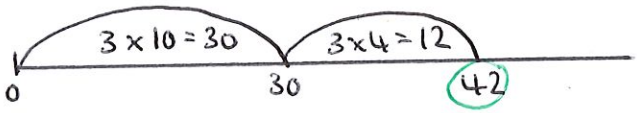


7
14
21
28
35

Y3: Partition 2-digit numbers and jump in multiples of 10, applying times table knowledge.

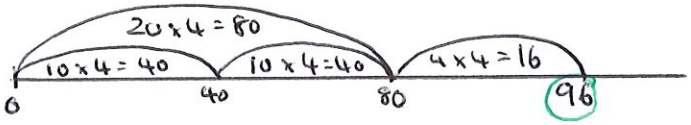
14 x 3 = □

10 4
(x3) + (x3)



24 x 4 = □

20 4
10 10 (x4)
(x4) (x4)



- Encourage pupils to apply to multiples of 10

Y4: Secure the grid method, and then column method for 2- and 3-digit numbers by 1-digit numbers.

x	200	40	3
3	600	120	9
			= 729

```
  24
x  3
----
  12   (3 x 4)
  60   (3 x 20)
----
  72   (+)
```

```
  24
x  3
----
  72
  +
```

- Record carried digits under the line. Cross out once added.

Y5: Secure the formal column method for a 4-digit number multiplied by a 1-digit number.

```
  2436
x    6
------
 14616
  2 2 3
```

- Record carried digits under the line. Cross out once added.

Y6: Secure the column method, for up to 4-digit numbers multiplied by 2-digit numbers plus.

```
  243
x  22
------
  486
+4860
------
 5346
  + +
```

```
  5976
x   48
--------
 47808
+23904 0
--------
286848
  +
```

- Record carried digits neatly under the line. Cross out once added.

Division

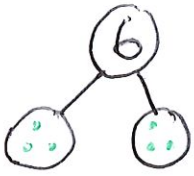
YR: Halve to solve problems, by sharing and grouping, using practical resources and visuals.

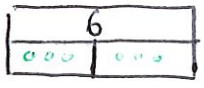
Cut the pizza in half to share with a friend.

3 friends share 6 apples equally.

Half of 4 = ☐

Y1: Explore sharing and grouping using sharing circles and arrays. Some children may be able to begin to represent this on the number line alongside arrays.

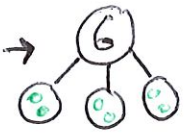




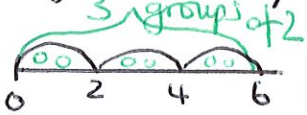
Half $\left(\frac{1}{2}\right)$ of 6 = ☐

6 ÷ 2 = ☐

SHARING ← → GROUPING

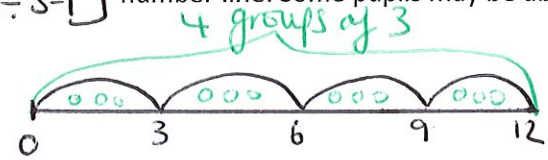


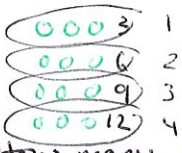
6 ÷ 2 = ☐ (Groups of 2 - how many fit inside?)



Y2: Grouping becomes the focus, considering e.g. "how many 3s fit inside 12?". Count up on a number line. Some pupils may be able to apply their times table knowledge.

12 ÷ 3 = ☐



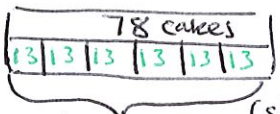


- Understand use of arrays for both multiplication and for division

KS2: Apply skills to a variety of problems; understand that the *representation/interpretation* of a problem might be *sharing*, but you can use a *grouping strategy* to work it out (commutativity).

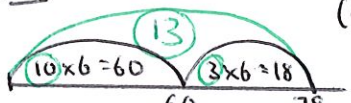
E.g. Mrs Bun shares 78 cakes between 6 customers. How many cakes will they get each?

Representation



(sharing)

78 ÷ 6 = ☐ Calculation



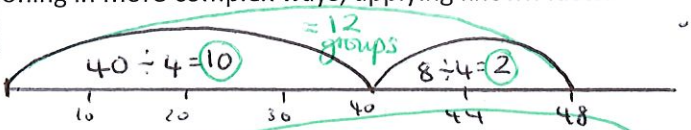
(grouping) is more efficient)

=13 each

Y3: First partition 2-digit numbers into tens and ones to count up in groups, applying known facts. Progress to partitioning in more complex ways, applying known facts.

48 ÷ 4 = ☐

40 8

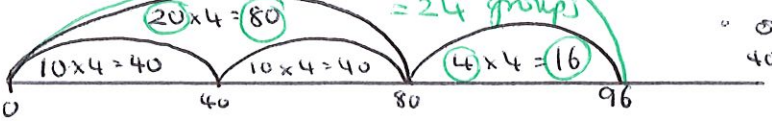


- or record as
  4 × 10 = 40 / 4 × 2 = 8

96 ÷ 4 = ☐

80 16

40 40



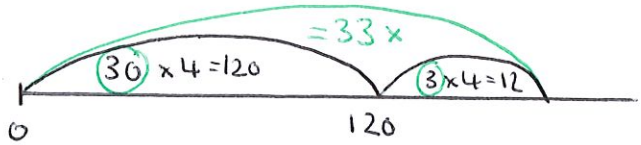
- or record as
  40 ÷ 4 = 10 | 40 ÷ 4 = 10 |
  16 ÷ 4 = 4

Y4: Continue to partition numbers in more complex ways to aid mental division, using known facts. List known facts to help to chunk division calculations (1x, 2x, 5x, 10x).

132 ÷ 4 = ☐

120 12

40 40 40



1 × 4 = 4
2 × 4 = 8
5 × 4 = 20
10 × 4 = 40

30 × 4 = 120
3 × 4 = 12
132

- Check using the inverse and known facts

Y5: Secure the short division method.

$$\begin{array}{r} 056 \\ 4\overline{)2^{2}2^{2}4} \end{array}$$

1 x 4 = 4
2 x 4 = 8
5 x 4 = 20
10 x 4 = 40

Y6: Secure long division when dividing by a 2-digit number. List known / worked facts to help chunk division calculations (1x, 2x, 5x, 10x).

$$\begin{array}{r} 024 \\ 37\overline{)888} \\ 74\downarrow \\ \hline 148 \end{array}$$

1 x 37 = 37
2 x 37 = 74
5 x 37 = 185
10 x 37 = 370

$$\begin{array}{r} 0046 \\ 37\overline{)1^{6}7^{1}02} \\ 148\downarrow \\ \hline 0222 \\ \hline \end{array}$$